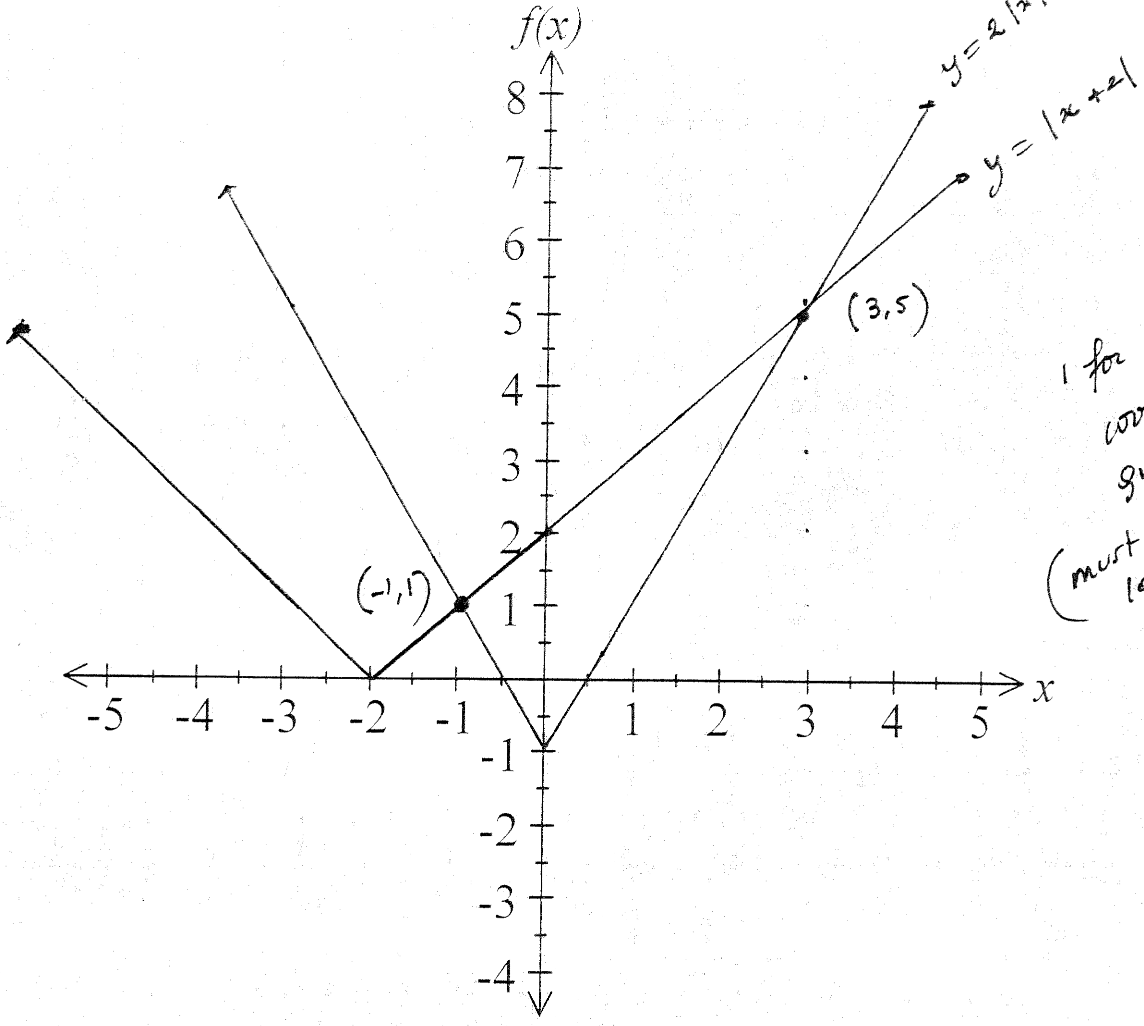
Section Two (calculator assumed)
Working time: 100 minutes.
Available marks: 80 marks

Question 1 [4 marks]

a) On the same set of axes draw the graphs of $y = 2|x| - 1$ and $y = |x + 2|$ [2]



2

1 for each correct graph (must be labelled) -1

(b) From your graphs obtain values of (x, y) which satisfy both equations [2]

2 solutions :

$x = -1$
$y = 1$ $(-1, 1)$

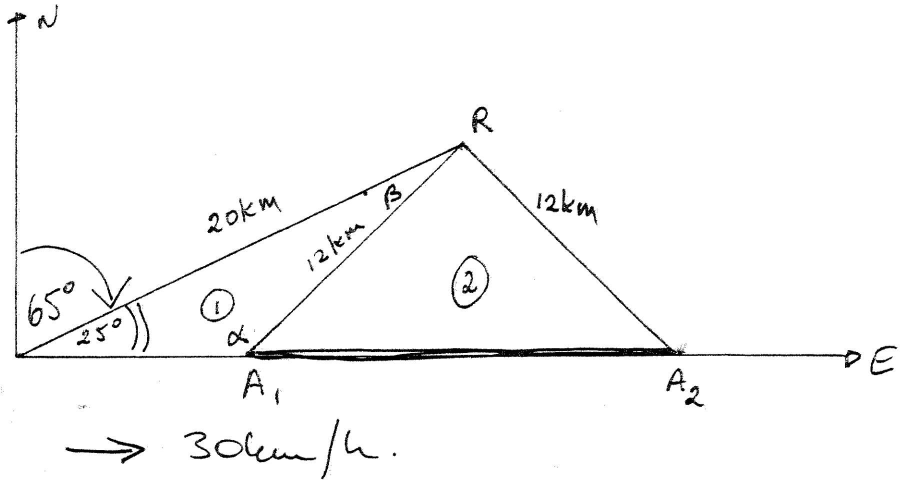
$x = 3$
$y = 5$ $(3, 5)$

-1 for any error.

4

Question 2 [5 marks]

At 3 pm, Alex is sailing due east at 30km/hr. He observes a reef on a bearing of 065° at a distance of 20 km. When will Alex be 12 km or less from the reef if he continues on this course? (Give answer correct to nearest minute).

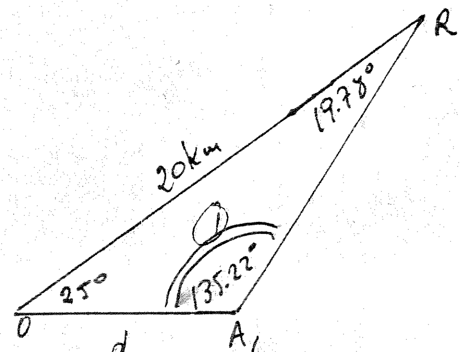


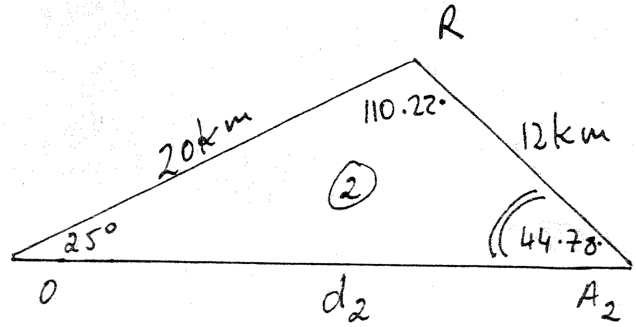
$$\frac{\sin \alpha}{20} = \frac{\sin 25°}{12}$$

AMBIGUOUS CASE

$\alpha = 44.78°$ → $\beta = 110.22°$

or $= 135.22°$ → $= 19.78°$





Using the sine rule:

$$\frac{d_1}{\sin 19.78°} = \frac{12}{\sin 25°}$$

$d_1 = 9.61\text{km}$ (2dp)

$$\text{time} = \frac{9.61}{30}$$

$= 0.320\text{h}$

$= 19.2\text{min}$

$= 19$ minutes

$$\frac{d_2}{\sin 110.22°} = \frac{12}{\sin 25°}$$

$d_2 = 26.64\text{km}$

$$\text{time} = \frac{26.64}{30}$$

$= 0.888\text{h}$

$\approx 53.28\text{min}$

$\approx$ 53 minutes

$\boxed{\therefore 3.19\text{pm} \leq t \leq 3.53\text{pm}}$

5

Alex will be 12km or less from the reef between 3.19pm and 3.53pm (inclusive)

Question 3. [**4 marks**]

A pyramid 16cm high has a square base with sides of 12cm.The apex A of the pyramid is directly above the centre C of the base PQRS.

(a) Calculate the length of its sloping edges. [2]

$CQ^2 = 6^2 + 6^2$

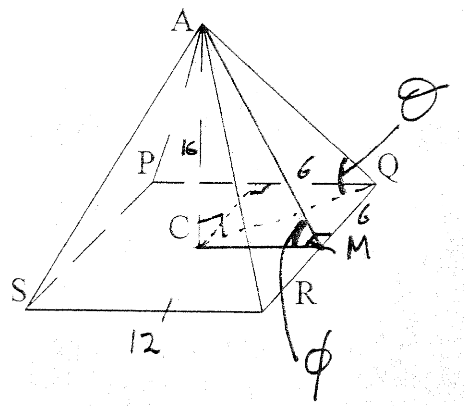
$CQ = \sqrt{72}$

$AQ^2 = 16^2 + 72$

$= 328$

$AQ = 2\sqrt{82}$

$AQ = \underline{\underline{18.11 cm}}$ (2dp)



(b) Calculate the angle made by the sloping edge with the base [1]

$\tan\theta = \frac{16}{\sqrt{72}}$

$\theta = \underline{\underline{62.06°}}$ (2dp)

$\theta \approx 62°$

(c) Calculate the angle made by a sloping face of the pyramid with its base. [1]

$\tan\phi = \frac{16}{6}$

$\phi = 69.44°$ (2dp)

$\simeq \underline{\underline{69°}}$

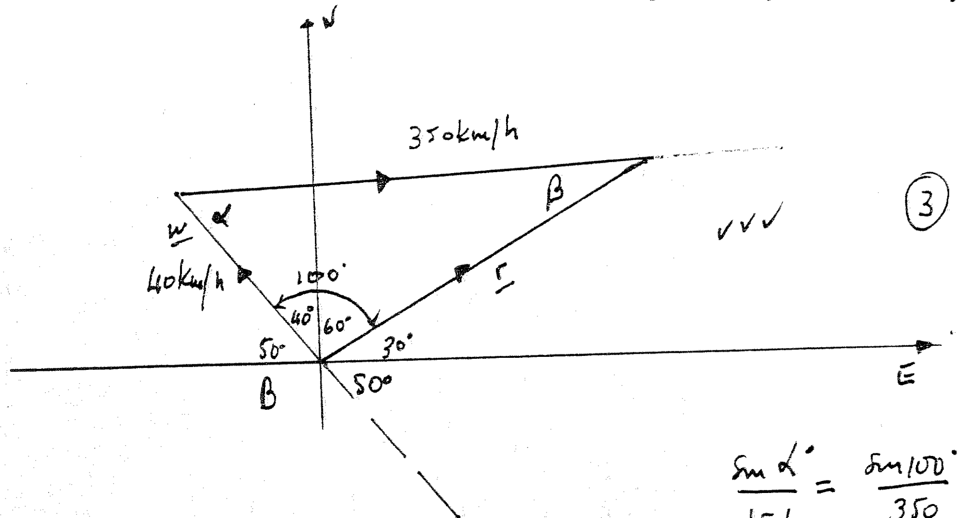
4

Question.4 [7 marks]

In still air Brad can fly his plane at a steady 350 km/hr. He needs to fly 810 km, to his holiday destination which is in a direction of 060° from his base.

The wind is blowing at 40 km/hr from 140°.

(a) In what direction must Brad fly to go directly to his holiday spot? [5]



✓✓✓ 3

$$\frac{\sin\alpha^\circ}{|\underset{\sim}{r}|} = \frac{\sin 100^\circ}{350} = \frac{\sin\beta^\circ}{40}$$

$\beta = 6.46^\circ$

$\alpha = 73.54^\circ$

Bearing $= 180^\circ - 40^\circ - 73.54^\circ$ 2

$= 066.46^\circ$

$\approx 066^\circ$

(b) How long does the trip take? [2]

$$\frac{|\underset{\sim}{r}|}{\sin 73.54^\circ} = \frac{350}{\sin 100^\circ}$$

$|\underset{\sim}{r}| = 340.83\text{km/h}$ 2dp

2

$$\text{Time} = \frac{810}{340.83}$$

$= 2.38h$

$= 2h\ 23min$

$2h\ 22\ min\ 36sec$

7

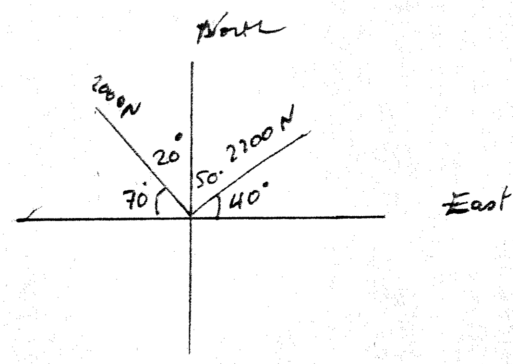
Question 5 **[5 marks]**

It looks as though the boat called "Hope" is still in trouble and is being dragged towards the same East-West reef by currents moving due South with a force of 3500N. Yet again, the rescue vessels called "Hero" and "Heroine" are trying to prevent a disaster by attaching rescue lines to the boat called "Hope".
Hero exerts a force of 2200N on a bearing of 050° and Heroine exerts a force of 2000N on a bearing of 340°.
Demonstrate the use of component vectors to determine the fate of the boat "Hope".

a) Find the resultant force exerted on Hope. *(in component form)* [3]

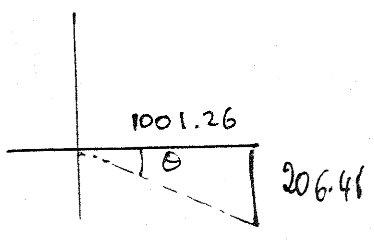
$\underset{\sim}{r} = 0\underset{\sim}{i} - 3500\underset{\sim}{j} + 2200\cos 40^\circ \underset{\sim}{i} + 2200\sin 40^\circ \underset{\sim}{j} - 2000\cos 70^\circ \underset{\sim}{i} + 2000\sin 70^\circ \underset{\sim}{j}$

$\underset{\sim}{r} = 1001.26\underset{\sim}{i} - 206.48\underset{\sim}{j}$



b) Its magnitude and direction. [2]

$|\underset{\sim}{r}| = 1022.33\text{ N}$



$\tan\theta = \dfrac{206.40}{1001.26}$

$\theta = 11.65^\circ$

Bearing required : $90^\circ + 11.65^\circ$

$= 101.65^\circ$

$\simeq 102^\circ$

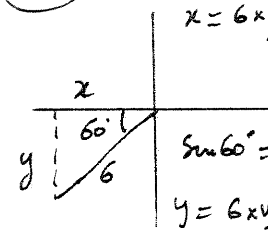
/5

Question 6 **[7 marks]**

(a) If the vector **a** has a polar angle of $\frac{4\pi}{3}$ and a magnitude of 6 metres, write **a** in the form **ai** +b**j** [2]

$\cos 60^\circ = \frac{x}{6}$

$x = 6 \times \frac{1}{2} = 3$



$\sin 60^\circ = \frac{y}{6}$

$y = 6 \times \frac{\sqrt{3}}{2}$

$y = 3\sqrt{3}$

$a\underset{\sim}{i} + b\underset{\sim}{j} = -3\underset{\sim}{i} - 3\sqrt{3}\,\underset{\sim}{j}$ ✓✓

5.196...

(b) If **a** = -3**i** + 4**j**, **b** = **i** + 2**j** and **c** = 2**i** – **j**, determine each of the following

(i) A vector parallel to **c** and twice as long [1]

$\underset{\sim}{v} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$

(ii) The magnitude of **a** [1]

$= \sqrt{9+16}$

$= 5$ units

(iii) The unit vector in the same direction as **c** [1]

$|\underset{\sim}{c}| = \sqrt{5}$

$\hat{\underset{\sim}{c}} = \frac{2}{\sqrt{5}}\underset{\sim}{i} - \frac{j}{\sqrt{5}}$

$= \frac{2}{\sqrt{5}}\underset{\sim}{i} - \frac{1}{\sqrt{5}}\underset{\sim}{j}$

0.89 ~~[illegible]~~ 0.45

or $= \frac{1}{\sqrt{5}}(2\underset{\sim}{i} - \underset{\sim}{j})$

(iv) A vector parallel to **a** with three times the magnitude of **c** [2]

$|\underset{\sim}{a}| = 5$ units

$\hat{\underset{\sim}{a}} = -\frac{3}{5}\underset{\sim}{i} + \frac{4}{5}\underset{\sim}{j}$ ✓ $(3|\underset{\sim}{c}| = 3\sqrt{5})$

vector required : $\underset{\sim}{v} = 3\sqrt{5}\left(-\frac{3}{5}\underset{\sim}{i} + \frac{4}{5}\underset{\sim}{j}\right)$ ✓

$= -\frac{9\sqrt{5}}{5}\underset{\sim}{i} + \frac{12\sqrt{5}}{5}\underset{\sim}{j}$

$-4.02\underset{\sim}{i} + 5.37\underset{\sim}{j}$

Question.7 **[8marks]**

Airports A and B are such that $\overline{AB}=(500i+1200j)$ km. An aircraft is to be flown directly from A to B.
In still air the aircraft can maintain a steady speed of 425 km/h.
There is a wind blowing with velocity $(24i+7j)$ km/h.
Find:

a) The velocity vector, in the form $(ai+bj)$ km/h. the pilot should set so that this velocity, together with the wind causes the plane to travel directly from A to B? Give your answer correct to 2 decimal places. [6]

$$a\underline{i} + b\underline{j} + 24\underline{i} + 7\underline{j} = \lambda(500\underline{i} + 1200\underline{j}) \quad \lambda > 0 \checkmark$$

✓ ① $a+24 = 500\lambda$
✓ ② $b+7 = 1200\lambda$
✓ ③ $a^2+b^2 = 425^2$

enter the 3 simultaneous equations in graphics calc to obtain: (2dp)

$a = 145.34$, $b = 399.42$ or $-\cancel{181.25}$ $\lambda > 0$

$\lambda = 0.33868$

$\therefore \quad \underline{v} = 145.34\underline{i} + 399.42\underline{j}$ ✓✓

ALGEBRA ① ÷ ②

$$\frac{5}{12} = \frac{a+24}{7+b}, \quad b = \frac{12a+253}{5}$$

$b = 2.4a + 50.6$ sub in ③

$$a^2 + (2.4a + 50.6)^2 = 180625$$

$$6.76a^2 + 242.88a - 178064.64 = 0$$

a and b as before.

b) The time taken to travel from A to B. [2]

$$\text{time} = \frac{1}{\lambda} = \frac{1}{0.33868}$$

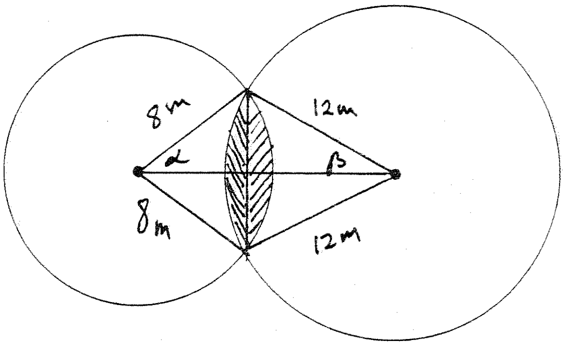
$= 2.95$ hours

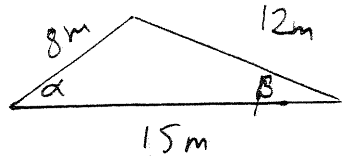
$= 2$ h 57 min 10 sec

$\approx$ 2h 57 min. ✓✓

Question 8 [7 marks]

Find the area common to two circles which have their centres 15m apart. One of the circles has an 8m radius and the other has a radius of 12m.





$\cos\alpha = \dfrac{8^2+15^2-12^2}{2\times8\times15}$

$\alpha = 0.92208$ RAD ✓

$2\alpha = 1.8442$ RAD

$\cos\beta = \dfrac{15^2+12^2-8^2}{2\times15\times12}$

$\beta = 0.56006$ RAD ✓

$2\beta = 1.1201$ RAD

Area of segment $= \frac{1}{2}\times 8^2(1.8442 - \sin 1.8442)$ ✓

$= 28.2029 m^2$ ✓

$A_{segment} = \frac{1}{2}\times 144(1.1201 - \sin 1.1201)$ ✓

$= 15.8368 m^2$ ✓

TOTAL AREA $= 44.0397 m^2$

$\approx \underline{\underline{44 m^2}}$ ✓

7

Question.9 [4marks]

The population of a town grows according to the rule $P = P_0(1.2)^t$ where t is in years

(a) Find the percentage increase in the population per year. [1]

20%

(b) How long does it take for the population to triple? [3]

$3P_0 = P_0(1.2)^t$

$3 = (1.2)^t$

$\log 3 = t \log 1.2$

$t = \frac{\log 3}{\log 1.2}$

$t = 6.025$

$\therefore$ it takes 6 years for the population to triple

Question 10 [4 marks]

Given that $_C\mathbf{r}_A = 4\mathbf{i} - 5\mathbf{j}$ and the position vector of C is $5\mathbf{i} + 6\mathbf{j}$, find how far and in what direction is A from O.

$\vec{OC} = 5i + 6j$

$_C r_A = 4i - 5j = \vec{AC}$

$\vec{AC} = \vec{AO} + \vec{OC}$

$4i - 5j = \vec{AO} + 5i + 6j$

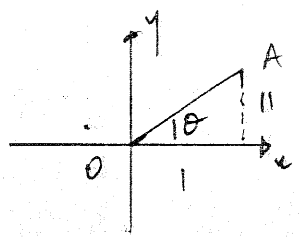
$\vec{AO} = 4i - 5j - 5i - 6j$

$\vec{AO} = -i - 11j$

$\vec{OA} = i + 11j$

$\vec{OA} = \sqrt{122} = 11.04$ units (2dp)

4



$\tan\theta = \frac{11}{1}$

$\theta = 84.81^\circ$

Direction required : $90 - 84.81^\circ$
$= 005.19^\circ$
$\approx 005.2^\circ$
$\approx 005^\circ$

Question 11 [4 marks]

The velocities (in m/s), of three moving objects P, Q, R are $5\mathbf{i} + 4\mathbf{j}$, $-\mathbf{i} + 7\mathbf{j}$ and $9\mathbf{i} - 2\mathbf{j}$ respectively.

a) Find the velocity of P relative to Q. [1]

$_P v_Q = v_P - v_Q$

$= \begin{pmatrix}5\\4\end{pmatrix} - \begin{pmatrix}-1\\7\end{pmatrix}$

$= \begin{pmatrix}6\\-3\end{pmatrix}$

$_P v_Q = 6i - 3j$

b) In what direction and with what speed is Q moving relative to R. [3]

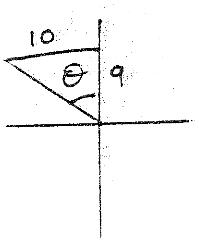
$_Q v_R = v_Q - v_R$

$= \begin{pmatrix}-1\\7\end{pmatrix} - \begin{pmatrix}9\\-2\end{pmatrix}$

$= \begin{pmatrix}-10\\9\end{pmatrix}$

$_Q v_R = -10i + 9j$

$|_Q v_R| = \sqrt{181}$ m/s $= 13.45$ m/s



$\tan\theta = \frac{10}{9}$

$\theta = 48.01^\circ$

Bearing $= 312^\circ$

8

Question 12 **[6 marks]**

Given that $f(x)=\dfrac{1}{x+1}$, find:

a) $f(\dfrac{1}{x})$ *in simplified form* [1]

$$= \frac{1}{\frac{1}{x}+1}$$

$$= \frac{1}{\frac{1+x}{x}}$$

$$f(\tfrac{1}{x}) = \frac{x}{1+x}$$

b) x such that $f(x)= f(\dfrac{1}{x})$ [2]

$$\frac{1}{x+1} = \frac{x}{1+x} \checkmark \qquad x \neq -1$$

$$1+x = x(x+1)$$

$$\cancel{1+x} = x^2+\cancel{x}$$

$$x^2 = 1$$

$$x = \pm 1 \qquad x \neq -1$$

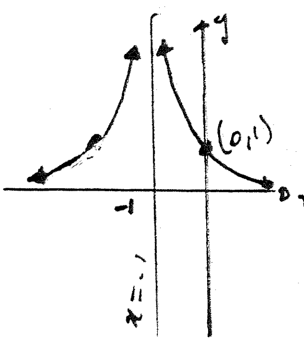
$$x = +1 \checkmark$$

Must give 2 answers and must reject one.

-1

c) The natural domain and range of g given that $g(x)=[f(x)]^2$ [3]

$$g(x) = \frac{1}{(x+1)^2} \checkmark \quad 1$$



Do: $\{x \in R : x \neq -1\}$ ✓ 1

Ra: $\{y \in R : y > 0\}$ ✓ 1

6

Question 13 [8 marks]

Given that f(x) = x-1, g(x)= $\frac{1}{x}$, find the largest possible domain for

a) f(x) so that g[f(x)] is a function. [2]

$R \to \boxed{f(x) = x-1} \to R \qquad x \neq 0 \to \boxed{g(x) = \frac{1}{x}} \to y \neq 0$

$y \neq 0$

$0 \neq x-1$

$x \neq 1$

the range of F(x) is not contained within the domain of g(x) so g[f(x)] does not exist

For g[f(x)] to exist the domain of f(x) must exclude $x = 1$

Domain of f(x): $\{x \in R : x \neq 1\}$

b) g(x) so that f[g(x)] is a function. [2]

$x \neq 0 \to \boxed{g(x) = \frac{1}{x}} \to y \neq 0 \qquad R \to \boxed{f(x) = x-1} \to R$

The range of g(x) is included in the domain of f(x) so f[g(x)] exists

The largest possible domain for g(x) is $\{x \in R : x \neq 0\}$

c) Using the restricted domain, state the rule, domain and range for each composite function. [4]

$g \circ f(x) = \frac{1}{x-1}$

Domain : $\{x \in R : x \neq 1\}$

Range : $\{y \in R : y \neq 0\}$

$f \circ g(x) = \frac{1}{x} - 1$

$= \frac{1-x}{x}$

Do : $\{x \in R : x \neq 0\}$

Ra : $\{y \in R : y \neq -1\}$

8

Question 14 [7 marks]

For the function $f(x) = (x+4)^2$,

a) Explain why f(x) cannot have an inverse function for its natural domain.[2]

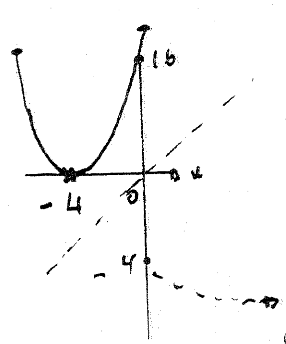
$f(-1) = (-1+4)^2 = 9$

$f(-7) = (-7+4)^2 = 9$

f(x) is a many to one function, therefore it has no inverse ✓ ✓ (2)

$y \geq 0$

$y = f(x)$



b) Find the largest possible domain for f(x) consisting only of negative numbers so that f(x) has an inverse. [1]

$f^{-1}(x)$ exists if $x \leq -4$

Domain required : $\{x \in R : x \leq -4\}$ ✓ (1)

c) For the domain in (b), find the rule for the inverse of f(x). [2]

$y = (x+4)^2$

$x = (y+4)^2$ (1) for method

$\pm\sqrt{x} = y + 4$

$y = -4 \pm \sqrt{x}$

$\boxed{f^{-1}(x) = -4 - \sqrt{x}}$ (1) for answer

d) For the domain in (b), find the domain and range for the inverse of f(x). [2]

Do : $\{x \in R, x \geq 0\}$ ✓ (2)

Ra : $\{y \in R, y \leq -4\}$ ✓

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